Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

1 - 8 Polar Form
Represent in polar form and graph in the complex plane as in Fig. 325.

1. 1 + I

```
Clear["Global`*"]
```

```
z = 1 + ⅈ
```

```
1 + ⅈ
```

```
(*ComplexToPolar[z_]/;z∈Complexes:=Abs[z] Exp[ⅈ Arg[z]]*)
```

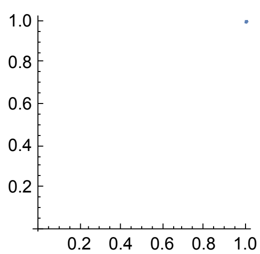
```
dd = Abs[z]; ee = Arg[z];
```

```
ComplexToPolar[dd_, ee_] := HoldForm[dd (Cos[ee] + ⅈ Sin[ee])]
```

```
ComplexToPolar[dd, ee]
```

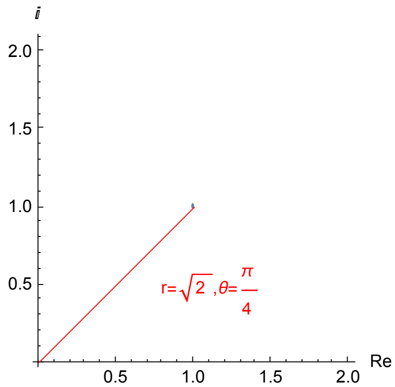
$$\sqrt{2}\left(\mathrm{Cos}\left[\frac{\pi}{4}\right] + i\,\mathrm{Sin}\left[\frac{\pi}{4}\right]\right)$$

$$\mathrm{PolarPlot}\left[\sqrt{2},\ \left\{\theta,\ \frac{\pi}{4},\ \frac{\pi}{4\,(.999)}\right\},\ \mathrm{ImageSize} \to 130\right]$$



An extra plot thrown in (above) just to show something in polar coordinates.

```
ListPlot[FromPolarCoordinates[{{dd, ee}}],
 AspectRatio → Automatic, ImageSize → 200, AxesLabel → {Re, I},
 Epilog → {Red, Line[{{0, 0}, {1, 1}}], Text["r=√2,θ=π/4", {1.1, 0.45}]}]
```



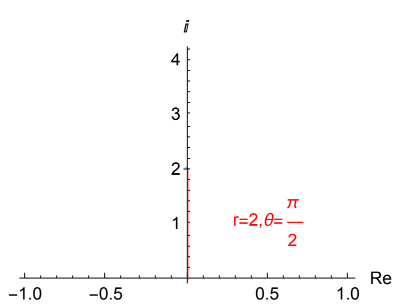
I reluctantly convert away from polar coordinates to cartesian, due to a shortcoming in Mathematica plotting capabilities, or to my ignorance.

3. 2 I, -2 I

```
z = 2 ⅈ
2 ⅈ

dd = Abs[z]; ee = Arg[z];

ComplexToPolar[dd_, ee_] := HoldForm[dd (Cos[ee] + ⅈ Sin[ee])]

ComplexToPolar[dd, ee]
```

$2\left(\text{Cos}\left[\frac{\pi}{2}\right] + i\,\text{Sin}\left[\frac{\pi}{2}\right]\right)$

```
ListPlot[FromPolarCoordinates[{{dd, ee}}],
 AspectRatio → .7, ImageSize → 200, AxesLabel → {Re, I},
 Epilog → {Red, Line[{{0, 0}, {0, 2}}], Text["r=2,θ=π/2", {0.5, 1}]}]
```

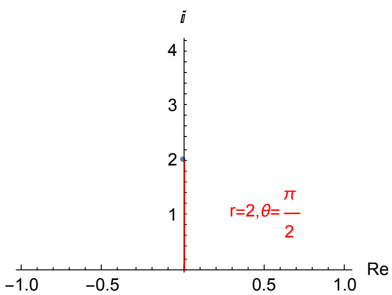



```
z = -2 ⅈ
-2 ⅈ
```

```
dd = Abs[z]; ee = Arg[z];
ComplexToPolar[dd_, ee_] := Hold[dd (Cos[ee] + ⅈ Sin[ee])]
ComplexToPolar[dd, ee]
```

$\text{Hold}\left[2\left(\text{Cos}\left[-\frac{\pi}{2}\right]+ⅈ\,\text{Sin}\left[-\frac{\pi}{2}\right]\right)\right]$

```
ListPlot[FromPolarCoordinates[{{dd, ee}}],
 AspectRatio → .7, ImageSize → 200, AxesLabel → {Re, I},
 Epilog → {Red, Line[{{0, 0}, {0, 2}}], Text["r=2,θ=π/2", {0.5, 1}]}]
```



5. $\dfrac{\sqrt{2}+ⅈ/3}{-\sqrt{8}-2ⅈ/3}$

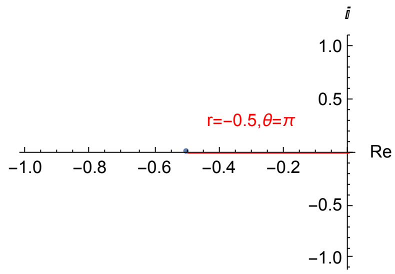
$$z = \frac{\sqrt{2}+ⅈ/3}{-\sqrt{8}-2ⅈ/3}$$

$$\frac{\frac{ⅈ}{3}+\sqrt{2}}{-\frac{2ⅈ}{3}-2\sqrt{2}}$$

```
dd = Abs[z]; ee = Arg[z];
ComplexToPolar[dd_, ee_] := HoldForm[dd (Cos[ee] + ⅈ Sin[ee])]
ComplexToPolar[dd, ee]
```

$\frac{1}{2}\left(\text{Cos}[\pi]+ⅈ\,\text{Sin}[\pi]\right)$

```
ListPlot[FromPolarCoordinates[{{dd, ee}}],
 AspectRatio → .7, ImageSize → 200, AxesLabel → {Re, I}, Epilog →
  {Red, Line[{{0, 0}, {-0.5, 0}}], Text["r=-0.5,θ=π", {-0.3, 0.3}]}]
```



7. $1 + \frac{1}{2} \pi i$

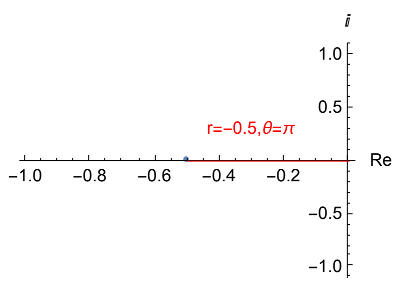
$z = 1 + \frac{1}{2} \pi i$

$1 + \frac{i \pi}{2}$

```
dd = Abs[z]; ee = Arg[z];
ComplexToPolar[dd_, ee_] := HoldForm[dd (Cos[ee] + i Sin[ee])]
ComplexToPolar[dd, ee]
```

$\sqrt{1 + \frac{\pi^2}{4}} \left(\text{Cos}\left[\text{ArcTan}\left[\frac{\pi}{2}\right]\right] + i\,\text{Sin}\left[\text{ArcTan}\left[\frac{\pi}{2}\right]\right]\right)$

```
ListPlot[FromPolarCoordinates[{{dd, ee}}],
 AspectRatio → .7, ImageSize → 200, AxesLabel → {Re, I}, Epilog →
  {Red, Line[{{0, 0}, {-0.5, 0}}], Text["r=-0.5,θ=π", {-0.3, 0.3}]}]
```



9 - 14 Principal Argument
Determine the principal value of the argument and graph it as in Fig. 325.

9. - 1 + I

In Mathematica the **Arg** function always returns a value between -π and π, so it is okay to just barge ahead with the calculation.

```
z = -1 + i
```

-1 + i

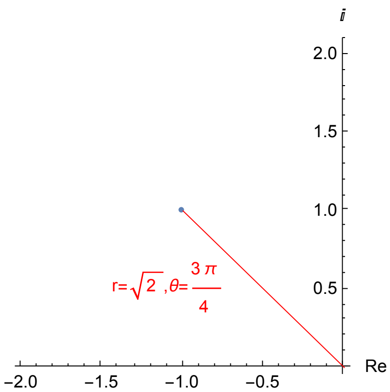
```
dd = Abs[z]
```

$\sqrt{2}$

```
ee = Arg[z]
```

$\frac{3\pi}{4}$

```
ListPlot[FromPolarCoordinates[{{dd, ee}}], AspectRatio → 1,
 ImageSize → 200, AxesLabel → {Re, I}, PlotRange → Automatic, Epilog →
  {Red, Line[{{0, 0}, {-1, 1}}], Text["r=√2,θ=3π/4", {-1.1, 0.5}]}]
```



11. 3 ± 4 I

```
z1 = 3 + 4 i; z2 = 3 - 4 i;
dd = Abs[z1]
```

5

```
ee = Arg[z1]
```

ArcTan$\left[\frac{4}{3}\right]$

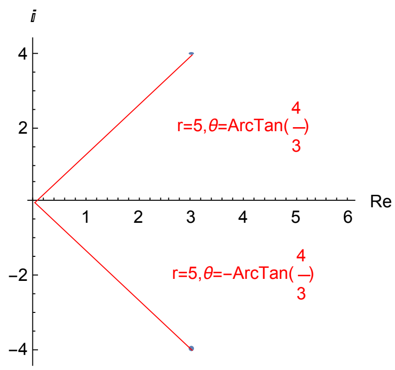
```
dd2 = Abs[z2]
```

5

```
ee2 = Arg[z2]
```

$-\text{ArcTan}\left[\frac{4}{3}\right]$

```
ListPlot[FromPolarCoordinates[{{dd, ee}, {dd2, ee2}}], AspectRatio → 1,
 ImageSize → 200, AxesLabel → {Re, I}, PlotRange → Automatic,
 Epilog → {Red, Line[{{0, 0}, {3, 4}}], Line[{{0, 0}, {3, -4}}],
   Text["r=5,θ=ArcTan(4/3)", {4, 2}], Text["r=5,θ=-ArcTan(4/3)", {4, -2}]}]
```



13. $(1 + i)^{20}$

```
z = (1 + i)^20
```

-1024

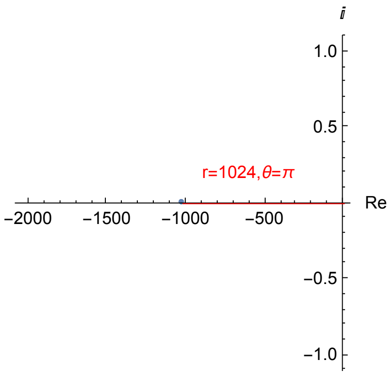
```
dd = Abs[z]
```

1024

```
ee = Arg[z]
```

π

```
ListPlot[FromPolarCoordinates[{{dd, ee}}], AspectRatio → 1,
 ImageSize → 200, AxesLabel → {Re, I}, PlotRange → Automatic, Epilog →
  {Red, Line[{{0, 0}, {-1024, 0}}], Text["r=1024,θ=π", {-600, 0.2}]}]
```



15 - 18 Conversion to x + iy

15. $3\left(\text{Cos}\left[\frac{1}{2}\pi\right] - i\,\text{Sin}\left[\frac{1}{2}\pi\right]\right)$

$$z = 3\left(\text{Cos}\left[\frac{1}{2}\pi\right] - i\,\text{Sin}\left[\frac{1}{2}\pi\right]\right)$$

$-3\,i$

17. $\sqrt{8}\left(\text{Cos}\left[\frac{1}{4}\pi\right] + i\,\text{Sin}\left[\frac{1}{4}\pi\right]\right)$

$$z = \sqrt{8}\left(\text{Cos}\left[\frac{1}{4}\pi\right] + i\,\text{Sin}\left[\frac{1}{4}\pi\right]\right)$$

$2 + 2\,i$

As can be seen from the two cells above, Mathematica does this section through automatic simplification.

21 - 27 Roots

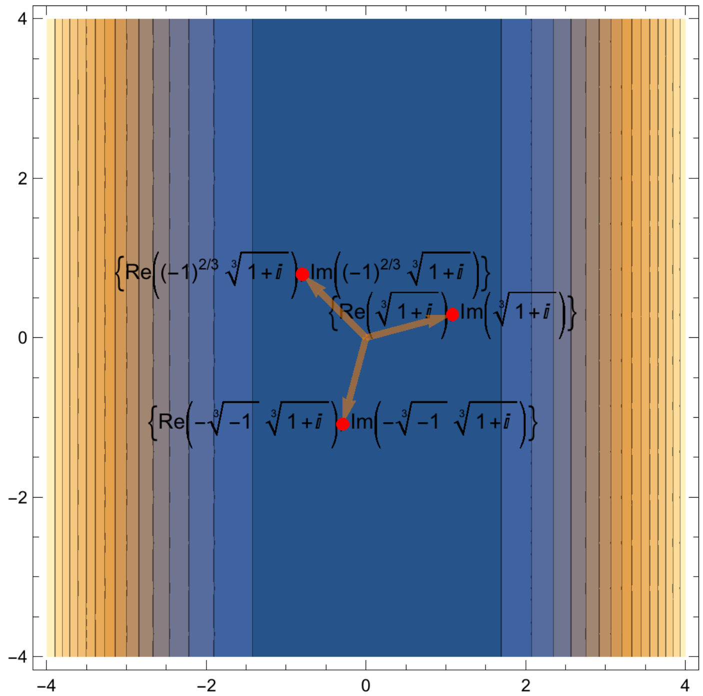
Find and graph all roots in the complex plane.

21. $\sqrt[3]{1 + i}$

$$\text{pts} = (\{\text{Re}@\#, \text{Im}@\#\}\ \&\ /@\ (x\ /.\ \text{Solve}[x^3 == 1 + i]))$$

$$\{\{\text{Re}[(1+i)^{1/3}], \text{Im}[(1+i)^{1/3}]\},$$
$$\{\text{Re}[-(-1)^{1/3}(1+i)^{1/3}], \text{Im}[-(-1)^{1/3}(1+i)^{1/3}]\},$$
$$\{\text{Re}[(-1)^{2/3}(1+i)^{1/3}], \text{Im}[(-1)^{2/3}(1+i)^{1/3}]\}\}$$

```
Show[ContourPlot[Abs[x³ - (1 + ⅈ)], {x, -4, 4}, {y, -4, 4}, Contours → 15],
 Graphics[{{Style[Text[#, #], 11] & /@ #},
     {Opacity[.5], Orange, Thickness[.01], Arrow[{{0, 0}, #}] & /@ #},
     {Red, PointSize[.02], Point@#}}, Axes → True, Frame → True,
    PlotRangePadding → 1, AspectRatio → Automatic] &@pts]
```



The above sketch, which is a little confusing, applies to the answer, and its precursor can be found in the Mathematica documentation on the page "Plot Complex Roots".

```
jj = Solve[x³ - 1 - ⅈ == 0, x]
```

$\{\{x \to (1 + i)^{1/3}\}, \{x \to -(-1)^{1/3} (1 + i)^{1/3}\}, \{x \to (-1)^{2/3} (1 + i)^{1/3}\}\}$

In order to make the equation easier to work with, I cubed both sides.

```
x /. jj
```

$\{(1 + i)^{1/3}, -(-1)^{1/3} (1 + i)^{1/3}, (-1)^{2/3} (1 + i)^{1/3}\}$

```
ComplexExpand[x³ - 1 - ⅈ /. jj]
```

{0, 0, 0}

The above cell demonstrates that all of Mathematica's roots work.

```
mine1 = N[(1 + ⅈ)^(1/3)]
```

1.08422 + 0.290515 ⅈ

The green cell above matches the text answer for one root, shown by answer below, ans1.

```
ans1 = N[2^(1/6) (Cos[π/12] + ⅈ Sin[π/12])]
```

1.08422 + 0.290515 ⅈ

```
ComplexExpand[    (1.0842150814913512` + 0.29051455550725136` ⅈ)^3 - 1 - ⅈ]
```

-1. + ⅈ (-1. + 1.   ) + 1.

In the above yellow cell the text's first answer, ans1, is proved to be a valid root.

```
mine2 = N[-(-1)^(1/3) (1 + ⅈ)^(1/3)]
```

-0.290515 - 1.08422 ⅈ

The green cell above matches a root found by the s.m. in a cyan cell, sm3, which does NOT match a text answer.

```
ans2 = N[2^(1/6) (Cos[9 π/12] + ⅈ Sin[π/12])]
```

-0.793701 + 0.290515 ⅈ

```
ComplexExpand[
    (-0.7937005259840997` + 0.29051455550725136` ⅈ)^3 - 1 - ⅈ]
```

-1. + ⅈ (-1. + 0.524519   ) - 0.299038

In the above brown cell a text answer, ans2, fails to prove as root.

```
sm2 = N[2^(1/6) (Cos[9 π/12] + ⅈ Sin[9 π/12])]
```

-0.793701 + 0.793701 ⅈ

```
mine3 = N[(-1)^(2/3) (1 + ⅈ)^(1/3)]
```

-0.793701 + 0.793701 ⅈ

The green cell above matches a root found by the s.m. in a cyan cell, sm2, which does NOT match a text answer.

```
ans3 = N[2^(1/6) (Cos[17 π/12] + ⅈ Sin[π/12])]
```

-0.290515 + 0.290515 ⅈ

```
ComplexExpand[    (-0.2905145555072514` + 0.2905145555072514` ⅈ)^3 - 1 - ⅈ]
```

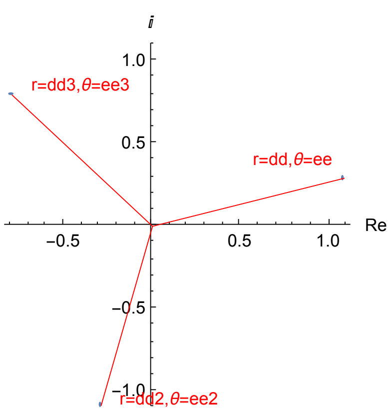
-1. + ⅈ (-1. + 0.0490381   ) + 0.0490381

$$sm3 = N\left[\sqrt[6]{2}\left(\text{Cos}\left[\frac{17\pi}{12}\right] + i\,\text{Sin}\left[\frac{17\pi}{12}\right]\right)\right]$$

-0.290515 - 1.08422 i

In the above brown cell a text answer, ans3, fails to prove as root. The answers of the s.m. in this case show that the text answer omits a vital “k” factor. For example, where ‘17’ shows in two places in the cyan cell above, it applies to only one place in the text answer.

```
dd = Abs[ComplexExpand[(1 + i)^(1/3)]];

ee = Arg[ComplexExpand[(1 + i)^(1/3)]];

cc = FromPolarCoordinates[{dd, ee}];

dd2 = Abs[ComplexExpand[-(-1)^(1/3) (1 + i)^(1/3)]];

ee2 = Arg[ComplexExpand[ -(-1)^(1/3) (1 + i)^(1/3)]];

cc2 = FromPolarCoordinates[{dd2, ee2}];

dd3 = Abs[ComplexExpand[(-1)^(2/3) (1 + i)^(1/3)]];

cc3 = FromPolarCoordinates[{dd3, ee3}];

ee3 = Arg[ComplexExpand[(-1)^(2/3) (1 + i)^(1/3)]];

ListPlot[FromPolarCoordinates[{{dd, ee}, {dd2, ee2}, {dd3, ee3}}],
 AspectRatio → 1.05, ImageSize → 200,
 AxesLabel → {Re, I}, PlotRange → {-1.1, 1.1},
 Epilog → {Red, Line[{{0, 0}, {cc[[1]], cc[[2]]}}],
   Text["r=dd,θ=ee", {0.8, 0.4}], Line[{{0, 0}, {cc3[[1]], cc3[[2]]}}],
   Text["r=dd3,θ=ee3", {-0.4, 0.85}], Line[
    {{0, 0}, {cc2[[1]], cc2[[2]]}}], Text["r=dd2,θ=ee2", {0.1, -1.05}]}]
```



The above sketch resembles the one in the s.m.

23. $\sqrt[3]{216}$

```
Clear["Global`*"]
```

```
jj = Solve[x^3 - 216 == 0, x]
```

$\{\{x \to 6\}, \{x \to -6\,(-1)^{1/3}\}, \{x \to 6\,(-1)^{2/3}\}\}$

```
x /. jj
```

$\{6, -6\,(-1)^{1/3}, 6\,(-1)^{2/3}\}$

```
ComplexExpand[x^3 - 216 /. jj]
```

{0, 0, 0}

The above cell demonstrates that all of Mathematica's roots work.

```
ComplexExpand[-6 (-1)^(1/3)]
```

$-3 - 3\,i\,\sqrt{3}$

```
ComplexExpand[6 (-1)^(2/3)]
```

$-3 + 3\,i\,\sqrt{3}$

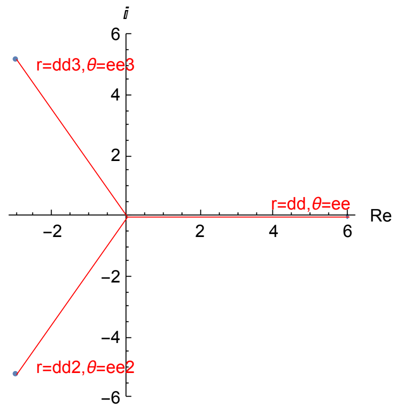
The roots in the green cell above match the answer in the text.

```
dd = Abs[ComplexExpand[6]];
ee = Arg[ComplexExpand[6]];
cc = FromPolarCoordinates[{dd, ee}];
```

```
dd2 = Abs[ComplexExpand[-6 (-1)^(1/3)]];
ee2 = Arg[ComplexExpand[-6 (-1)^(1/3)]];
cc2 = FromPolarCoordinates[{dd2, ee2}];
```

```
dd3 = Abs[ComplexExpand[6 (-1)^(2/3)]];
ee3 = Arg[ComplexExpand[6 (-1)^(2/3)]];
cc3 = FromPolarCoordinates[{dd3, ee3}];
```

```
ListPlot[FromPolarCoordinates[{{dd, ee}, {dd2, ee2}, {dd3, ee3}}],
 AspectRatio → 1.05, ImageSize → 200, AxesLabel → {Re, I},
 PlotRange → {-6, 6}, Epilog → {Red, Line[{{0, 0}, {cc[[1]], cc[[2]]}}],
   Text["r=dd,θ=ee", {5, 0.4}], Line[{{0, 0}, {cc3[[1]], cc3[[2]]}}],
   Text["r=dd3,θ=ee3", {-1.1, 5}], Line[{{0, 0}, {cc2[[1]], cc2[[2]]}}],
   Text["r=dd2,θ=ee2", {-1.1, -5}]}]
```



25. $\sqrt[4]{i}$

```
Clear["Global`*"]
```

```
jj = Solve[x^4 - i == 0, x]
```

$\{\{x \to -(-1)^{1/8}\}, \{x \to (-1)^{1/8}\}, \{x \to -(-1)^{5/8}\}, \{x \to (-1)^{5/8}\}\}$

```
x /. jj
```

$\{-(-1)^{1/8}, (-1)^{1/8}, -(-1)^{5/8}, (-1)^{5/8}\}$

```
ComplexExpand[x^4 - i /. jj]
```

{0, 0, 0, 0}

The above cell demonstrates that all of Mathematica’s roots work.

$\text{hre}[k\_] = \text{Cos}\left[\frac{\pi}{8} + \frac{k\pi}{2}\right] + i\,\text{Sin}\left[\frac{\pi}{8} + \frac{k\pi}{2}\right]$

$\text{Cos}\left[\frac{\pi}{8} + \frac{k\pi}{2}\right] + i\,\text{Sin}\left[\frac{\pi}{8} + \frac{k\pi}{2}\right]$

```
tab = Table[hre[k], {k, 0, 3}]
```

$\left\{\text{Cos}\left[\frac{\pi}{8}\right] + i\,\text{Sin}\left[\frac{\pi}{8}\right],\ i\,\text{Cos}\left[\frac{\pi}{8}\right] - \text{Sin}\left[\frac{\pi}{8}\right],\ -\text{Cos}\left[\frac{\pi}{8}\right] - i\,\text{Sin}\left[\frac{\pi}{8}\right],\ -i\,\text{Cos}\left[\frac{\pi}{8}\right] + \text{Sin}\left[\frac{\pi}{8}\right]\right\}$

```
Simplify[
 Table[ComplexExpand[k^4 - ⅈ, {k, {Cos[π/8] + ⅈ Sin[π/8], ⅈ Cos[π/8] - Sin[π/8],
     -Cos[π/8] - ⅈ Sin[π/8], -ⅈ Cos[π/8] + Sin[π/8]}}], {k, {Cos[π/8] + ⅈ Sin[π/8],
    ⅈ Cos[π/8] - Sin[π/8], -Cos[π/8] - ⅈ Sin[π/8], -ⅈ Cos[π/8] + Sin[π/8]}}]]
{0, 0, 0, 0}
```

The above cell demonstrates that all of the text's roots work. According to the Rouché theorem there should be exactly four roots, so these roots must be the same as Mathematica's, right? To demonstrate, I will convert one of them:

```
TrigToExp[Cos[π/8] + ⅈ Sin[π/8]]
```

$e^{\frac{i\pi}{8}}$

```
Simplify[%]
```

$(-1)^{1/8}$

On the basis of the above there is green cell above, though the format of individual answers is not exactly like the answers in the text.

```
dd = Abs[ComplexExpand[-(-1)^(1/8)]];
ee = Arg[ComplexExpand[-(-1)^(1/8)]];
cc = FromPolarCoordinates[{dd, ee}];

dd2 = Abs[ComplexExpand[(-1)^(1/8)]];
ee2 = Arg[ComplexExpand[(-1)^(1/8)]];
cc2 = FromPolarCoordinates[{dd2, ee2}];
```

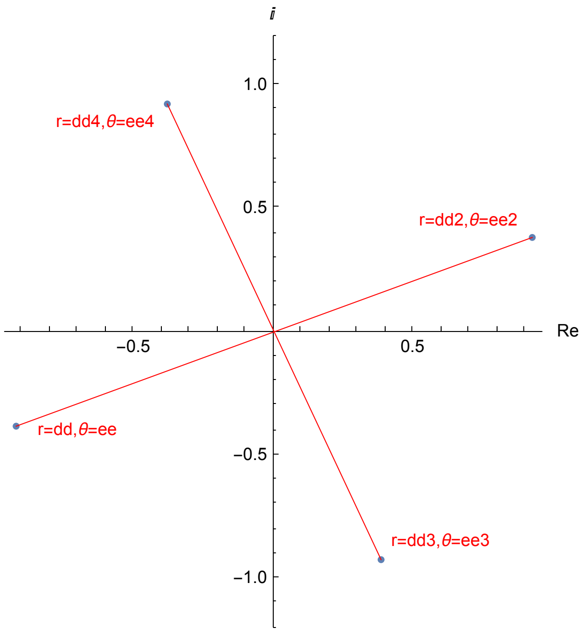
N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating $Log[Cos[\frac{\pi}{8}]^2 + Sin[\frac{\pi}{8}]^2]$. ≫

```
dd3 = Abs[ComplexExpand[-(-1)^(5/8)]];
ee3 = Arg[ComplexExpand[-(-1)^(5/8)]];
cc3 = FromPolarCoordinates[{dd3, ee3}];
```

N::meprec : Internal precision limit $MaxExtraPrecision = 50.` reached while evaluating $Log[Cos[\frac{\pi}{8}]^2 + Sin[\frac{\pi}{8}]^2]$. ≫

```
dd4 = Abs[ComplexExpand[(-1)^(5/8)]];
ee4 = Arg[ComplexExpand[(-1)^(5/8)]];
cc4 = FromPolarCoordinates[{dd4, ee4}];
```

```
ListPlot[FromPolarCoordinates[
  {{dd, ee}, {dd2, ee2}, {dd3, ee3}, {dd4, ee4}}], AspectRatio → 1.1,
 ImageSize → 300, AxesLabel → {Re, I}, PlotRange → {-1.2, 1.2},
 Epilog → {Red, Line[{{0, 0}, {cc[[1]], cc[[2]]}}],
   Text["r=dd,θ=ee", {-0.7, -0.4}], Line[{{0, 0}, {cc3[[1]], cc3[[2]]}}],
   Text["r=dd3,θ=ee3", {0.6, -0.85}],
   Line[{{0, 0}, {cc4[[1]], cc4[[2]]}}],
   Text["r=dd4,θ=ee4", {-0.6, 0.85}], Line[
    {{0, 0}, {cc2[[1]], cc2[[2]]}}], Text["r=dd2,θ=ee2", {0.7, 0.45}]}]
```



27. $\sqrt[5]{-1}$

```
Clear["Global`*"]
```

```
jj = Solve[x^5 + 1 == 0, x]
```

$\{\{x \to -1\}, \{x \to (-1)^{1/5}\}, \{x \to -(-1)^{2/5}\}, \{x \to (-1)^{3/5}\}, \{x \to -(-1)^{4/5}\}\}$

```
x /. jj
```

$\{-1, (-1)^{1/5}, -(-1)^{2/5}, (-1)^{3/5}, -(-1)^{4/5}\}$

```
ComplexExpand[x^5 + 1 /. jj]
```

{0, 0, 0, 0, 0}

The above cell demonstrates that all of Mathematica's roots work.

$\text{tr1} = \text{Cos}\left[\frac{\pi}{5}\right] + i\,\text{Sin}\left[\frac{\pi}{5}\right];$

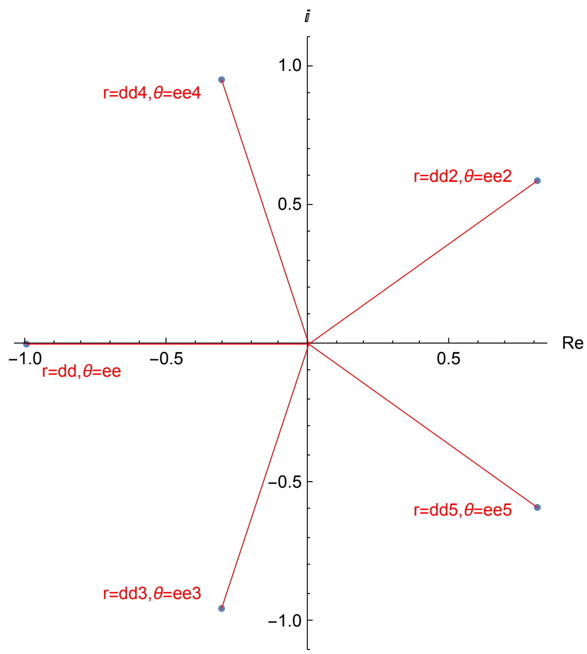
```
FullSimplify[ComplexExpand[tr1]]
```

```
(-1)^(1/5) (* jj[[2]]*)

tr2 = Cos[π/5] - ⅈ Sin[π/5];

FullSimplify[ComplexExpand[tr2]]

-(-1)^(4/5) (* jj[[5]]*)

tr3 = Cos[3 π/5] - ⅈ Sin[3 π/5];

FullSimplify[ComplexExpand[tr3]]

-(-1)^(2/5)  (* jj[[3]]*)

tr4 = Cos[3 π/5] + ⅈ Sin[3 π/5];

FullSimplify[ComplexExpand[tr4]]

(-1)^(3/5) (* jj[[4]]*)

tr5 = -1; (* jj[[1]]*)
```

The above five tests find the text’s roots equal to Mathematica's.

```
dd = Abs[ComplexExpand[-1]];
ee = Arg[ComplexExpand[-1]];
cc = FromPolarCoordinates[{dd, ee}];

dd2 = Abs[ComplexExpand[(-1)^(1/5)]];
ee2 = Arg[ComplexExpand[(-1)^(1/5)]];
cc2 = FromPolarCoordinates[{dd2, ee2}];

dd3 = Abs[ComplexExpand[-(-1)^(2/5)]];
ee3 = Arg[ComplexExpand[-(-1)^(2/5)]];
cc3 = FromPolarCoordinates[{dd3, ee3}];

dd4 = Abs[ComplexExpand[(-1)^(3/5)]];
ee4 = Arg[ComplexExpand[(-1)^(3/5)]];
cc4 = FromPolarCoordinates[{dd4, ee4}];

dd5 = Abs[ComplexExpand[-(-1)^(4/5)]];
ee5 = Arg[ComplexExpand[-(-1)^(4/5)]];
cc5 = FromPolarCoordinates[{dd5, ee5}];
```

```
ListPlot[FromPolarCoordinates[
   {{dd, ee}, {dd2, ee2}, {dd3, ee3}, {dd4, ee4}, {dd5, ee5}}],
  AspectRatio → 1.15, ImageSize → 300, AxesLabel → {Re, I},
  PlotRange → {-1.1, 1.1},
  Epilog → {Red, Line[{{0, 0}, {cc[[1]], cc[[2]]}}],
    Text["r=dd,θ=ee", {-0.8, -0.1}], Line[{{0, 0}, {cc3[[1]], cc3[[2]]}}],
    Text["r=dd3,θ=ee3", {-0.55, -0.9}],
    Line[{{0, 0}, {cc4[[1]], cc4[[2]]}}],
    Text["r=dd4,θ=ee4", {-0.55, 0.9}],
    Line[{{0, 0}, {cc5[[1]], cc5[[2]]}}],
    Text["r=dd5,θ=ee5", {0.55, -0.6}], Line[
     {{0, 0}, {cc2[[1]], cc2[[2]]}}], Text["r=dd2,θ=ee2", {0.55, 0.6}]}]
```



28 - 31 Equations
Solve and graph the solutions.

29. $z^2 + z + 1 - i == 0$

```
Clear["Global`*"]
```

```
jj = Solve[z^2 + z + 1 - i == 0, z]
```

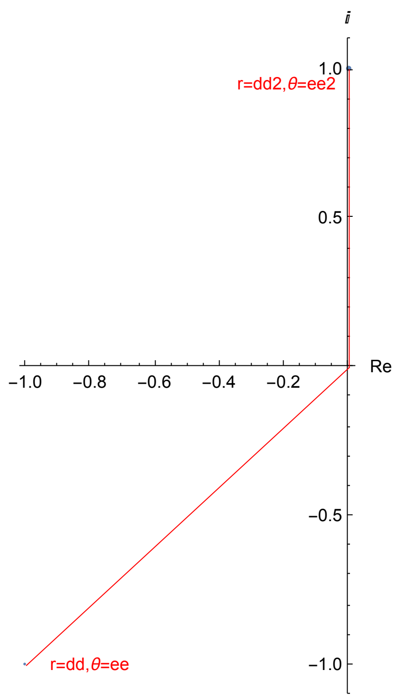
```
{{z → -1 - i}, {z → i}}
```

```
z /. jj
{-1 - i, i}
```

```
ComplexExpand[z² + z + 1 - i /. jj]
{0, 0}
```

The above cell demonstrates that both of Mathematica's roots work. The green cell shows them equal to the text's.

```
dd = Abs[ComplexExpand[-1 - i]];
ee = Arg[ComplexExpand[-1 - i]];
cc = FromPolarCoordinates[{dd, ee}];

dd2 = Abs[ComplexExpand[i]];
ee2 = Arg[ComplexExpand[i]];
cc2 = FromPolarCoordinates[{dd2, ee2}];

ListPlot[FromPolarCoordinates[{{dd, ee}, {dd2, ee2}}],
 AspectRatio → 1.95, ImageSize → 200,
 AxesLabel → {Re, I}, PlotRange → {-1.1, 1.1},
 Epilog → {Red, Line[{{0, 0}, {cc[[1]], cc[[2]]}}],
   Text["r=dd,θ=ee", {-0.8, -1}], Line[{{0, 0}, {cc2[[1]], cc2[[2]]}}],
   Text["r=dd2,θ=ee2", {-0.19, 0.95}]}]
```



31. $z^4 - 6 i z^2 + 16 == 0$

```
Clear["Global`*"]

jj = Solve[z⁴ - 6 i z² + 16 == 0, z]
{{z → -2 - 2 i}, {z → -1 + i}, {z → 1 - i}, {z → 2 + 2 i}}
```

```
z /. jj
```

```
{-2 - 2 i, -1 + i, 1 - i, 2 + 2 i}
```

```
ComplexExpand[z^4 - 6 i z^2 + 16 /. jj]
```

```
{0, 0, 0, 0}
```

The above cell demonstrates that all of Mathematica’s roots work. The green cell shows them equal to the text’s.

```
dd = Abs[ComplexExpand[-2 - 2 i]];
ee = Arg[ComplexExpand[-2 - 2 i]];
cc = FromPolarCoordinates[{dd, ee}];
```

```
dd2 = Abs[ComplexExpand[-1 + i]];
ee2 = Arg[ComplexExpand[-1 + i]];
cc2 = FromPolarCoordinates[{dd2, ee2}];
```

```
dd3 = Abs[ComplexExpand[1 - i]];
ee3 = Arg[ComplexExpand[1 - i]];
cc3 = FromPolarCoordinates[{dd3, ee3}];
```

```
dd4 = Abs[ComplexExpand[2 + 2 i]];
ee4 = Arg[ComplexExpand[2 + 2 i]];
cc4 = FromPolarCoordinates[{dd4, ee4}];
```

```
ListPlot[
 FromPolarCoordinates[{{dd, ee}, {dd2, ee2}, {dd3, ee3}, {dd4, ee4}}],
 AspectRatio → 1, ImageSize → 270, AxesLabel → {Re, I},
 PlotRange → {-2.1, 2.1}, Epilog → {Red,
   Line[{{0, 0}, {cc[[1]], cc[[2]]}}], Text["r=dd,θ=ee", {-1.3, -1.9}],
   Line[{{0, 0}, {cc4[[1]], cc4[[2]]}}],
   Text["r=dd4,θ=ee4", {1.35, 1.95}],
   Line[{{0, 0}, {cc3[[1]], cc3[[2]]}}],
   Text["r=dd3,θ=ee3", {1.55, -0.8}],
   Line[{{0, 0}, {cc2[[1]], cc2[[2]]}}],
   Text["r=dd2,θ=ee2", {-0.3, 0.95}]}]
```